Math 3 **3.5 Applications of Exponentials and Logs** Unit 3

SWBAT solve real-life application problems involving half-life and exponential growth and decay.

What in REAL LIFE can give us these kinds of functions?

Exponential Growth: When a quantity increases at a fixed percent (and increasing rate) over time

Exponential Decay: When a quantity decreases at a fixed percent (and increasing rate) over time

***To calculate exponential growth and decay, we use the formula:

$$\boxed{y = a(1 \pm r)^t}$$

A = initial amt (principal)
t = time
r = rate (as deci)
y = ending amt

Solving for Outcomes (y): Exponential Growth and Decay

Example 1: The population of a small town in North Carolina is 4,000 and is growing at a rate of 3% per year.

a) Write an exponential function to represent this data.

$y = 4000(1+0.03)^t$

b) What will the population of the town be in 10 years?

$y = 4000(1.03)^{10}$

$y = 5376$ people

Example 2: James purchased a truck for $29,500. The value of the truck depreciates by 12% per year.

a) Write an exponential function to represent this data.

$y = 29500(1-0.12)^t$

b) What will be the value of the truck 8 years after the purchase?

$y = 29500(0.88)^8$

$y = \$10,609.22$

You Try! A computer engineer creates a software program worth $28,000. If the program depreciates at 35% each year, how much money will the program be worth in 4 years? Write and solve an exponential function that represents this data.

$y = 28000(1-0.35)^4$

$y = \$4998.18$

Solving for Rate (r): HIV in the United States (All information obtained from www.cdc.gov)

Example 3: In 1981, the Human Immunodeficiency Virus (HIV) was discovered. In 1987, it is estimated that 50,280 people in the United States were diagnosed with the infection. In 2000, that number grew to 264,405 people within the United States. Given this information, find the rate of growth for this infection.

$264405 = 50280(1+r)^{13}$

$5.26 = (1+r)^{13}$

$\sqrt[13]{5.26} = 1+r$

$1.136 = 1+r$

$r = 0.136 \times 100 = 13.6\%$

Example 4: Between 1988 and 1992, the number of HIV patients who died from HIV related causes was 181,212. In 2000, the number of HIV related deaths decreased to 59,807. Find the rate of decay for the number of deaths caused by HIV between 1988 and 2000.

$59807 = 181212(1-r)^{12}$

$0.33 = (1-r)^{12}$

$\sqrt[12]{0.33} = 1-r$

$1-r = 0.912$

$-r = -0.088$

$r = 0.088 \times 100 = 8.8\%$

Solving for Time (t): Money

Example 5: How long will it take \$30,000 to accumulate to \$110,000 in a trust that earns a 10% annual interest?

$110000 = 30000(1+0.10)^t$

$3.67 = (1.10)^t$

$\log 3.67 = (t)\log(1.1)$

$t = \dfrac{\log 3.67}{\log 1.1}$

$t = 13.6$ years

Example 6: How long does it take to double \$1000 at an annual interest rate of 6.35%?

$2000 = 1000(1+0.0635)^t$

$2 = (1.0635)^t$

$\log 2 = t\log 1.0635$

$t = \dfrac{\log 2}{\log 1.0635}$

$t = 11.3$ years

You Try! Currently, the National Debt is estimated at about \$16.69 trillion. If no more spending occurs, how long will it take for this number to reach \$20 trillion at 4.9% interest?

$20 = 16.69(1+0.049)^t$

$1.2 = (1.049)^t$

$\log 1.2 = t\log 1.049$

$t = \dfrac{\log 1.2}{\log 1.049}$

$t = 3.8$ years

Solving for the rate (r) given an equation:

Example 7: Does the function $y=295(0.72)^t$ represent exponential growth or decay? What is the rate?

$1-0.72 = 0.28$

$r = 28\%$ Decay

Example 8: Does the function $y=0.75(3.42)^t$ represent exponential growth or decay? What is the rate?

$3.42-1 = 2.42$

$r = 242\%$ Growth

You Try! Does the function $y=12(14.36)^t$ represent exponential growth or decay? What is the rate?

$14.36-1 = 13.36$

$r = 1336\%$ growth

You Try! Does the function $y=0.015(0.24)^t$ represent exponential growth or decay? What is the rate?

$1-0.24 = 0.76$

$r = 76\%$ decay

***To calculate continuously half-life, we use the formula:

Half-Life: $y=(a)\left(\frac{1}{2}\right)^{\left(\frac{time}{halflife}\right)}$

y = ending amt

a = initial amt

Example 9: Archaeologists use carbon-14, which has a half-life of 5730 years, to determine the age of artifacts in carbon dating.

a) Write the exponential decay function for a 24-mg sample.

$y = 24(0.5)^{t/5730}$

b) How much carbon-14 remains after 30 millennia? (Hint: 1 millennium = 1000 years)

$y = 24(0.5)^{30000/5730}$

$y = 0.64$ mg

You Try! Thallium-201 has a half-life of 73 hours. If 4.0 mg of thallium-201 disintegrates over a period of 6.0 days and 2 hours, how many mg of thallium-201 will remain?

HL = 73 hrs

t = 146 hrs

$y = 4(0.5)^{146/73}$

$y = 1$ mg